3.5

Equation Solving and Modeling

One to one properties:

For any exponential function $f(x) = b^x$,

- If $b^u = b^v$, then $u = v$.

For any logarithmic function $f(x) = \log_b x$,

- If $\log_b u = \log_b v$, then $u = v$.

Solving Exponential Equations:

Solve: $20\left(\frac{1}{2}\right)^{\frac{x}{3}} = 5$

$$20\left(\frac{1}{2}\right)^{\frac{x}{3}} = 5$$

$$\left(\frac{1}{2}\right)^{\frac{x}{3}} = \frac{1}{4}$$

$$\left(\frac{1}{2}\right)^{\frac{x}{3}} = \left(\frac{1}{2}\right)^2$$

$$\frac{x}{3} = 2 \qquad x = 6$$

See example 2 pg. 321.

Solving Logarithmic Equations:

Solve: $\log x^2 = 2$.

$\log x^2 = 2 \quad \rightarrow \quad 10^2 = x^2 \quad \rightarrow \quad x = 10$ or -10

See methods 1 & 3 also pg. 321- 322.

Solve: $\ln(3x - 2) + \ln(x - 1) = 2\ln x$.

$\ln(3x - 2) + \ln(x - 1) = 2\ln x$

$\ln[(3x - 2)(x - 1)] = \ln x^2$

$(3x - 2)(x - 1) = x^2$

$3x^2 - 3x - 2x + 2 = x^2$

$2x^2 - 5x + 2 = 0$

$(2x - 1)(x - 2) = 0$

$x = ½$ or $x = 2$ check domain $x \neq ½$ so $x = 2$ is the only solution.

We can also solve this by graphing, setting the original equation equal to 0 and finding the x-intercepts.

Solve: $\ln(3x - 2) + \ln(x - 1) = 2\ln x$.

$\ln(3x - 2) + \ln(x - 1) - 2\ln x = 0$

X intercept is at (2, 0).

Orders of Magnitude and Logarithmic Models

The common logarithm of a positive quantity is its **order of magnitude.**

Mercury is 57.9 billion meters from the sun.

Pluto is 5900 billion meters from the sun.

So Pluto's distance from the sun is 2 orders of magnitude greater than Mercury because it is 10^2 greater than Mercury's distance.

* A kilometer is 3 orders of magnitude longer than a meter.

* A dollar is 2 orders of magnitude greater than a penny.

Orders of magnitude are used to compare the severity of earthquakes and the acidity of chemical solutions

Comparing Earthquake Intensities:

$R = \log\frac{a}{T} + B$ where *a* = amplitude in micrometers (μm) or the vertical ground motion at the receiving station; T = period of the seismic wave in seconds; B accounts for the weakening of the seismic wave with increasing distance from the epicenter of the earthquake.

See example 5 pg. 324.

Comparing Chemical Acidity :

$$pH = -\log[H^+]$$

Where H+ is the hydrogen-ion concentration and pH is the measure of acidity used

See example 6 pg. 325.

Newton's Law of Cooling :

$T(t) = T_m + (T_o - T_m)\, e^{-kt}$

T_m = the temperature of the surrounding medium

T_o = initial temperature of the object.

See example 7 pg. 326.